

# Fourier Transform Profilometry in 3-Dimensions with Matlab programming

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**Abstract.** A Fourier transform method is used to process images recording with fringe projection technique. In this work, the fringe projection technique is tested in objects to obtain an image series, where each image is processing applying a Fast Fourier Transform algorithm. The image it is processing, enhancement and treating using Matlab programming. It is described the algorithm employed and implemented with matlab programming, the method to record the images, the stages of the experimental recording and how these images are processing to obtain a phase, and finally a recover the surface. The algorithm is tested in simple objects (spherical ball), showing that we have a good results.

**Key words:** Interferometry, Image processing, Fast Fourier Transform, Profilometry, Fringe projection.

## I. Introduction

Profilometry [1] is a non contact method that allows a fast and non destructive, non invasive inspection. There are many methods that use fringe projection and have been studied the last decades, among these methods there is the Fourier Transform Profilometry [2], using in this work, this technique present advantages

because only one image is used into the process of analysis. With a series of strips an object under test is illuminated and the characteristics of the surface are showing with the deformation of the strips, an image with fringes is recording. The surface shapes are recovered using Fourier transform, a filter in frequency domain and finally with the inverse Fourier transform process. Over the years, there has been an increase importance in development and applications of techniques for signal and image processing and some techniques are characterized with a parameter known as phase, in this area of the knowledge; signal magnitude is referred to as amplitude, whereas square magnitude is referred as intensity. In the present work, the phase parameter helps to image processing, related with amplitude and intensity.

This work described the Fourier transform procedure, the algorithm used and its implementation, to apply this Fourier transform method we used Matlab language programming. Experimental setups using fringe projection technique are described, and the obtained results with geometrical object are presented.

### **I.1 Fourier Transform Profilometry**

The method proposed by M. Takeda it is a computer- based technique for automatic 3-D shape measurement, a grating pattern projected onto the object surface is Fourier-transformed and processed in its spatial frequency domain as well as its space signal domain. This technique has a much higher sensitivity than the conventional moiré technique [3] and is capable of fully automatic distinction between a depression and elevation on the object surface.

## **II. Fast Fourier Transform Procedure. Phase Determination**

The phase is obtaining using the Fourier Transform Method, [2]. In optical measurements a fringe pattern can express as,

$$g(x, y) = a(x, y) + b(x, y) \cos[2\pi f_o x + \phi(x, y)], \quad (1)$$

where the phase  $\phi(x, y)$  contains the shape of an object under test, and  $a(x, y), b(x, y)$  represent variations in the background irradiance and contrast in the image, respectively. A fringe pattern of the form (1) is put into a computer by a CCD camera, image-sensing device that has enough resolution to satisfy the sampling-theory requirement, particularly in x-direction. The input fringe pattern is rewritten as follows,

$$g(x, y) = a(x, y) + c(x, y) \exp(2\pi i f_o x) + c^* x, y) \exp(-2\pi i f_o x) \quad (2)$$

Where  $c^*$  represent the complex conjugate, and

$$c(x, y) = \left(\frac{1}{2}\right)b(x, y)\exp[i\phi(x, y)] \tag{3}$$

Equation (2) is Fourier transformed with respect to  $x$ , using a fast-Fourier-transform (FFT) algorithm, and the result is

$$G(f, y) = A(f, y) + C(f - f_o, y) + C^*(f + f_o, y) \tag{4}$$

The capital letters denote the Fourier spectra;  $f$  is the spatial frequency in the  $x$ -direction. We use either the two spectra on the carrier, say  $C(f - f_o)$  and translated it by  $f_o$  on the frequency axis toward the origin to obtain  $C(f, y)$ . Note that the unwanted background variation  $a(x, y)$  has been filtered out this stage. Again, using the FFT algorithm, we compute the inverse Fourier transform of  $C(f, y)$  with respect to  $f$  and it is obtained  $c(f, y)$ . Then we calculate a complex logarithm of (3),

$$\log[c(x, y)] = \log\left[\frac{1}{2}b(x, y)\right] + i\phi(x, y) \tag{5}$$

Now we have the phase  $\phi(x, y)$  in the imaginary part completely separate from the unwanted amplitude variation  $b(x, y)$ . The process it is show in a diagram of the figure 1, where the algorithm wrap.m is sketched and the process order is indicated.

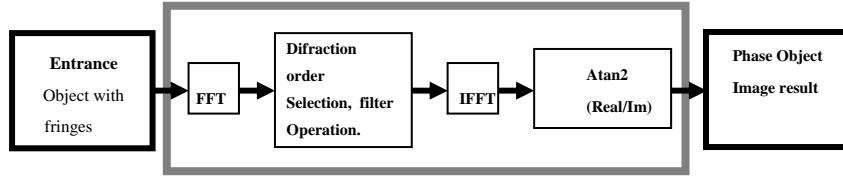


Fig. 1. Diagram of the wrap, unwrapping and phase recover algorithm, showing the stages and steps that are follow to image processing.

### II.1 Algorithm implementation

The algorithm was implemented using Matlab language programming [4], logic process follows these steps:

1. An image in fits extension it is read.
2. It is call a wrapping module, that follow the next steps:

- It is obtained the FFT2 of the input image, as result it delivers a complex value.
  - The former image is display in logarithm mode, it is obtained a matrix that allows to display the spectrum Fourier modes.
  - It is show the image with the modes, zero order and first one (positive and negative) order.
3. It is selected one mode, usually is taking the positive first order.
  4. It is call a filter (circle window).
  5. At this selected region, it is apply a FFTSHIF function.
  6. The result of the formal process, give us a matrix data that take the components of high frequencies to the edges.
  7. It is apply the FFT2 at these results.
  8. After this step it is come back to the spatial image
  9. It is separate and selected the real and imaginary part.
  10. It is apply each the ATAN2 function.
  11. Finally we get the phase of the image
  12. The Takeda method to unwrap the phase is applied.
  13. The final image it is display.

### III. Experiments

Some experiments were development to test the Fourier transform algorithms to image processing; an experimental setup presented in figure 2 is prepared to generate harmonic fringes programmed into the computer. This computer is used to control the fringe pattern program, fringe patterns are displayed using a Liquid Crystal Display (LCD) projector. It was displayed a fringe pattern created in Matlab programming with resolution of 25 pixels per period, it means, 1 line/12.5 pixel. The projection goes over an object that changes the straight fringes shape by object's topology. The physical fundamental parameters in the experimental setup are: object-camera distance, LCD- object distance.

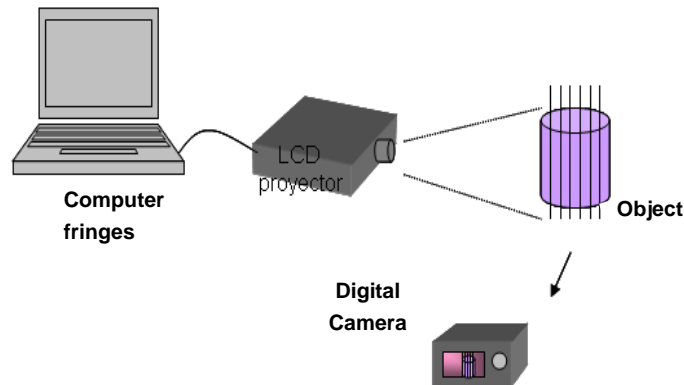
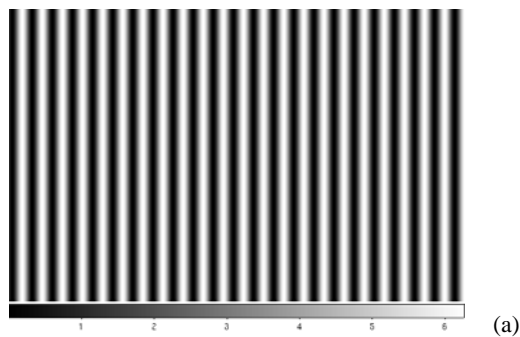


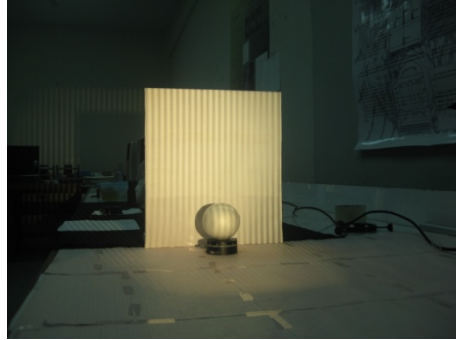
Fig. 2. Experimental Fringe projection set up. The fringes are display with a Liquid Crystal Display (LCD), the object under test is illuminated with a series of strips, controlled with a computer program, finally the image it is acquired with a digital camera.

### III.1 Experiments and results

It was taken images using the experimental setup described at figure 2; the images were acquiring using fringe projection techniques. The image to process has .fits extension; however, also we work with .jpg extension.

Images were acquired with a digital camera SD 1100IS, with 8.0 mega pixels, at jpg extension, as a part of the process, the image has .fits, extension, as a part of the process implemented at Matlab; besides, to manage the image, we used SAO IMAGE DS9 4.0b9 software. Figure 3 present a sequence of the process, starting from the fringe projection, through the final phase image resultant. At (a) it is presented a fringe pattern without any obstacle, as a reference image, next, at (b) we can see one angle of the experimental setup, showing a sphere with fringes over a background plane and over itself, a close up can see at (c), where it can appreciate the deformation generated by the sphere. At figure3 (d) is showing an image, as a result of processing, presenting a phase map image, we can notice some errors, as dark areas; after some more detailed processing, we can generated one (e) image with better results, along this image we indicating some contour lines and a grid in pixels.

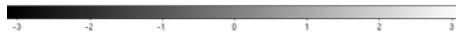




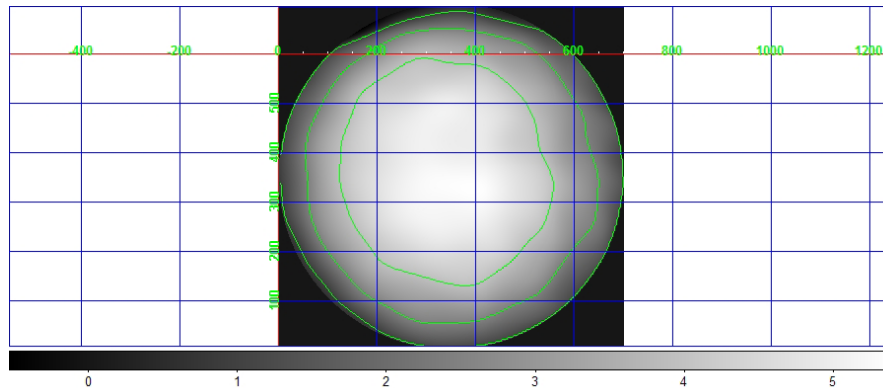
(b)



(c)



(d)



(e)

**Fig. 3.** Results generated by the fringe projection technique and image processing with Fourier transform. A sequence of the process is presented, at (a) we have the fringe pattern generated at the computer, (b) a sphere place into the experimental setup with fringe projection, look at the deformation at (c), follow at background straight line, that adopt the shape of the sphere. At (d) we present the result of the image processing, here we can observe the phase of the object. We can see at (e) Unwrapped phase generated.

## Conclusions

In the present work, we describe briefly the fringe profilometry technique, describing the experimental setup and the logic steps that follow a process, to get the image phase, such process used fast Fourier transform procedure. These logic processes were programming in Matlab language. We generated fringe patterns through computer programs, projected with a LCD projector; we obtained images that were processing, generating nice and quite good results to obtain the 3-dimensions topology of a sphere, as an object under test. With these results, we can continue improving the processing to expand the algorithm to automate all the experiments components [5] and expand the applications [6] of the fringe projection and Fourier transforms profilometry, for example, to vision system analysis in three dimensions.

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